Karunya University
(Karunya Institute of Technology and Sciences)
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End Semester Examinations - 2015-16 Even Semester - May 2016

15MA3010 Advanced Calculus

Set B **Time : 3 hrs**
Total Marks: 100

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1. (a). Let f be a increasing function defined on [a,b] and let $x_0, x_1, ..., x_n$ be n+1 points such that $a = x_0 < x_1 < ... < x_n = b$. Prove that the inequality
$$\sum_{k=1}^{n-1} [f(x_k+) - f(x_k-)] \le f(b) - f(a).$$ (Marks 10)

(b). If f is monotonic on [a,b], prove that the set of discontinuities of f is countable. (Marks 10)

OR

2. Assume that f and g are each of bounded variation on [a,b]. Prove that so are their sum, difference and product and also prove that $V_{f\pm g} \le V_f + V_g$ and $V_{f.g} \le AV_f + BV_g$ where $A = \sup\{|g(x) : x \in [a,b]|\}, B = \sup\{|f(x) : x \in [a,b]|\}$.

3. (a). If f∈R(α) and f∈R(β) on [a,b], prove that $f \in R(c_1\alpha + c_2\beta)$ on [a,b] and also prove that $\int_a^b fd(c_1\alpha + c_2\beta) = c_1\int_a^b fd\alpha + c_2\int_a^b fd\beta$. (Marks 10)

(b). Assume that c∈(a,b). If two of the three integrals in $\int_a^c fd\alpha + \int_c^b fd\alpha = \int_a^b fd\alpha$ exist prove that the third also exists. (Marks 10)

OR

4. (a). If f∈R(α) and g∈R(α) on [a,b], prove that $c_1f + c_2g \in R(\alpha)$ on [a,b] and also prove that $\int_a^b (c_1f + c_2g)d\alpha = c_1\int_a^b fd\alpha + c_2\int_a^b gd\alpha$.(Marks 10)

(b). If f∈R(α) on [a,b], prove that $\alpha \in R(f)$ on [a,b] and also prove
$$\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a).$$ (Marks 10)

5. (a). Assume that α is increasing on [a,b]. Prove that
   i) If P' is finer than P, then $U(P', f, \alpha) \le U(P, f, \alpha)$ and $L(P', f, \alpha) \ge L(P, f, \alpha)$.
   ii) For any two partitions P_1 and P_2, then $L(P_1, f, \alpha) \le U(P_2, f, \alpha)$. (Marks 10)

(b). Assume that α is increasing on [a,b]. Prove that $\underline{I}(f, \alpha) \le \bar{I}(f, \alpha)$. (Marks 10)

OR

6. Assume that α is of bounded variation on [a,b]. Let V(x) denote the total variation of α on [a,x] if $a<x\le b$, and let V(a) = 0. Let f defined and bounded on [a,b]. If $f\in R(\alpha)$ on [a,b], prove that $f \in R(V)$ on [a,b].

7. Let $\{f_n\}$ be a sequence of upper functions such that

i) $\{f_n\}$ increases almost everywhere on an interval I, and

ii) $\lim_{n\to\infty} \int_I f_n$ exists.

Prove that $\{f_n\}$ converges almost everywhere on I to a limit function f in U(I), and

$$\int_I f = \lim_{n\to\infty} \int_I f_n .$$

OR

8. Let $\{f_n\}$ be a sequence of Lebesgue-integrable functions on an interval I. Assume that

i) $\{f_n\}$ converges almost everywhere on I to a limit function f, and

ii) There is a nonnegative function g in L(I) such that, for all $n \ge 1$, $|f_n(x) \le g(x)|$ a.e. on I.

Prove that the limit function $f \in L(I)$, the sequence $\{\int_I f_n\}$ converges and

$$\int_I f = \lim_{n\to\infty} \int_I f_n .$$

9. State and prove Implicit function theorem.

Wishing you All the Best
